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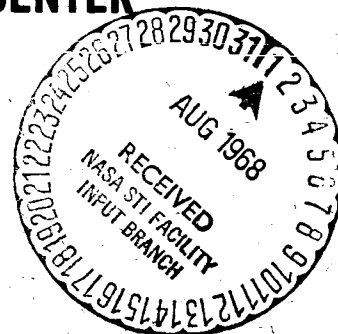
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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND



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ABSTRACT

The elements of the orbits of about 1000 earth satellites (as of April 1968) have been examined. The object was to determine those in resonance or near resonance with the earth's longitude gravity field which have the greatest promise for future satellite geodesy. Thirty-six resonant objects orbiting from one to fifteen times a day have been selected as having the greatest promise for improving knowledge of the geopotential. The main criterion used for the selection within each resonant frequency was the estimated strength of the strongest resonant perturbation based on a 1966 gravity field solution. By far the most favorable geodetic objects appear to be those with orbital frequencies of two revolutions a day. Among these are five objects which should be suffering resonant perturbations of from about 7000 to 400,000 kilometers along track, with periods of the order of years. But a large number of other quite strong resonant orbits with a good range of inclinations are also found to exist at frequencies of twelve and fourteen revolutions a day. Among the objects of 12 revs./day orbital frequency are five which should be suffering resonant perturbations of from about 0.5 to 10 kilometers along track, with periods of from 15 to 100 days. Among the objects of 14 revs./day orbital frequency are five which should be suffering resonant perturbations of from 0.7 to 80 kilometers along track with periods of from 21 to 250 days. As far as can be determined none of the thirty-six objects has yet been used to determine longitude components of the earth's gravity field. A scarcity of good resonant orbits is noted at four, five, six, seven, nine and ten revolutions a day.

A SURVEY OF EXISTING SATELLITES

IN RESONANT ORBITS FOR GEODETIC PURPOSES

INTRODUCTION

The stimulus for this survey was the intriguing question; of all the thousand or so existing satellite orbits, which of those in near resonance with the earth's rotation rate would be most useful for future geodetic investigations? More precisely, this extensive survey was undertaken once the ability was found to quickly calculate the so-called resonant beat period for near resonant (or commensurable) orbits from widely published¹ mean element specifications.²

While it is generally true that the most useful resonant orbits should be those with long beat periods, to allow for greater amplification of the relevant gravity effects, the essential tracking criterion is the actual perturbation itself. To assess the relative usefulness of existing orbits for future resonant satellite geodesy over the entire resonance spectrum, one would like an efficient method to estimate both beat period and perturbation amplitude.

Fortunately, Kaula's expansion of the geopotential³ enables one to calculate these off resonant or beat period perturbations quite readily from a straightforward first order integration of the Lagrange planetary equations. Such an integration will show the off resonant perturbation, in mean anomaly for example, to be sinusoidal for each relevant harmonic component, with a frequency that depends on the distance of the orbits mean motion from exact resonance for that component. The maximum mean anomaly acceleration, due to a particular

harmonic component, is a function of the semimajor axis, inclination and eccentricity of the orbit. Graphs of these component accelerations for the resonance spectrum with mean motions of from one to fifteen times a day, have already been tabulated.⁴ Using these graphs and the estimation of resonant beat period as calculated in Reference 2, one can quickly estimate the magnitudes of the linear forced oscillations in the along track position due to longitude terms in the geopotential.

Since 1965, quite a few low altitude geodetic satellites have been studied whose orbits have off resonant beat periods of from 2 to 10 days.^{5,6,7,8,9} In addition, two high altitude 12 hour orbits have been analyzed whose dominant near resonance beat periods have been between 100 and 150 days.^{10,11} Although there is no sharp division between the deep resonance and libration regimes (with pendulum characteristics) and the near or off resonance regimes which are characterized by linear perturbations, the simple beat period does provide a rough guide to these regimes. Off resonance, where the assumptions of simple forced linear oscillations are adequate, appears limited to orbits with beat periods less than about 100 days. Between beat periods of about 100 to perhaps 400 days, the orbit should be classified as near or deeply resonant whose perturbations are characteristic of a slowly circulating pendulum. Orbits with indicated beat periods over 400 days are probably in libration-like regimes. These resonant regime distinctions are examined in greater detail by Garfinkel¹² and Gedeon.¹³ Recently there has been published a list of about 50 objects in near resonant

orbits which includes those previously tracked for geodetic purposes as well as a number of new candidates for such tracking (Strange, et al.¹⁴). In Wagner² are found still more objects of geodetic interest in orbits with off resonant beat periods greater than 50 days. This report is designed to classify all of these, and other newly found near resonant orbits, in terms of the single important tracking criteria; the likely amplitude of the resonance perturbation due to the earth's longitude dependent gravity forces. This classification will show that there are far many more interesting near resonant objects existing that have never been tracked for geodetic purposes than have.

ANALYSIS

It is known (from Gedeon, et al.¹⁵) that for a perfectly resonant (or commensurable orbit) satellite, the acceleration of the mean anomaly due to a disturbing longitude harmonic component $V_{\ell_{mpq}}$ (in Kaula's form of the geopotential³) is given by:

$$\ddot{M}_{\ell_{mpq}} = - \frac{3m}{s} \mu \frac{a_e^\ell}{a^{\ell+3}} J_{\ell_m} F_{\ell_{mp}}(I) G_{\ell_{mpq}}(e) \begin{cases} -\sin & \ell-m \text{ even} \\ \cos & \ell-m \text{ odd} \end{cases} \Psi_{\ell_{mpq}}, \quad (1)$$

where s is the rational fraction number of revolutions the commensurate orbit satellite makes in a day, μ is the earth's Gaussian gravity constant, a_e is the mean equatorial radius of the earth, a is the orbit semimajor axis, J_{ℓ_m} is the unnormalized associated Legendre gravity harmonic coefficient of degree ℓ and order (or frequency) m ($m \neq 0$), $F_{\ell_{mp}}$ and $G_{\ell_{pq}}$ are inclination and eccentricity

functions (see Kaula³), and;

$$\Psi_{\ell mpq} = (\ell - 2p + q) M + (\ell - 2p) \omega + m(\Omega - \theta_e - \lambda_{\ell m}) \quad (2)$$

with the restrictions on the ℓ, m, p, q integral indeces that:

$$\ell - 2p + q = m/s$$

$$0 \leq p \leq \ell,$$

and

$$0 < m \leq \ell. \quad (3)$$

In Equation 2; ω, M and Ω are the Keplerian orbit elements, argument of perigee, mean anomaly and right ascension of the ascending node, θ_e is the hour angle of the Greenwich Meridian, and $\lambda_{\ell m}$ is the phase angle of the spherical harmonic of gravity whose amplitude is $J_{\ell m}$.

For near resonant orbits, Equation 1, which provides the pendulum analogy for the libration regime, while not exact, is a sufficiently good approximation of the disturbing acceleration to be useful for these resonant orbit classification purposes. The effects in the off resonant regime which it describes may be thought of as the echo of the very long period librational resonance effects in the same sense that the circulating pendulum is an echo of the behavior of the librating pendulum. When the off resonant beat period approaches one day, these resonant echo effects will be competing with another series of ordinary (or

non-resonant) longitude harmonic effects on the orbit of about one day period (due to the earth's rotation).

With the additional assumption that in the off or near resonance regime the longitude drift rates $\dot{\Psi}$ (or $\dot{\lambda}$, as in Wagner^{10,2}), and the orbit parameters I (inclination), e (eccentricity) and a (semimajor axis) on the right hand side of (1) do not change significantly, this equation can be written as:

$$\ddot{M} = C \left| \frac{-\sin}{\cos} \right| (\Psi_0 + \dot{\Psi}t) , \quad (4)$$

where;

$$C = \ddot{M}_{\text{maximum}} = - \frac{3m}{s} \mu \frac{a_e^\ell}{a^{\ell+3}} J_{\ell m} F_{\ell m p} (I) G_{\ell m p q} (e) . \quad (4a)$$

Equation 4 is the acceleration in a simple harmonic oscillator. The longitude rate $\dot{\Psi}$ is given by

$$\dot{\Psi} = (\ell - 2p + q) (\dot{M} + \dot{\omega}) + m (\dot{\Omega} - \dot{\theta}_e) - \dot{\omega}_q .$$

Using the resonance index selector $\ell - 2p + q = m/s$, these rates become:

$$\dot{\Psi}_{\ell m p q} = m \left\{ \frac{(\dot{M} + \dot{\omega})}{s} + \dot{\Omega} - \dot{\theta}_e - \frac{\dot{\omega}_q}{m} \right\} .$$

($\dot{\Psi}$ for near resonance will be close to zero.) In most cases, the last term on the right can be neglected. Then, these longitude rates are identical to m times the mean orbital longitude rate ($\dot{\lambda}$) used in Wagner² to derive the dominant beat

periods for the entire resonance spectrum. In a future development of this investigation, the neglected perigee rate term will be included to examine the fine structure of the spectrum for completeness.

In a harmonic oscillation of frequency $\dot{\Psi}$ and amplitude A, the amplitude of the acceleration (C in this problem) is simply $C = (\dot{\Psi})^2 A$. Thus $A = C/(\dot{\Psi})^2$.

In terms of the oscillation period T, $A = CT^2/(2\pi)^2$, since $\dot{\Psi}T = 2\pi$ radians.

In Douglas and Palmiter,⁴ values of C (maximum resonant accelerations of the mean anomaly) are plotted for the dominant resonance spectrum from S = 1 to 15 revolutions per day and I = 0° to 90° inclinations. The orbit eccentricity is fixed for each resonant period at the maximum allowable to yield a drag free perigee of about 750 km. In addition, the C values are based on a 1966 (combined satellite-surface gravimetric) geoid (Kaula¹⁶) which estimates $J_{\ell m}$ coefficients beyond $J_{12,12}$ by a rule of thumb that apparently underestimates some of the resonant coefficients at S = 13, 14 and 15 by as much as an order of magnitude. While the graphs of C values in Douglas and Palmiter⁴ are not as realistic as to actual orbit or gravity parameters as they might be, they do provide a basis for a reasonable first cut estimation of the perturbation amplitudes which can be expected on existing satellite orbits. The difference in eccentricity between the graphs and the existing orbits is never very large. In any case, the extended fine spectrum study will calculate exact C values based on the actual orbit parameters, a, e and I.

In Douglas and Palmiter,⁴ the C values are in units of degrees/day². In Wagner,² the off resonant beat periods BP_R are calculated (for assumed dominant effects where m = s) from the abbreviated elements in the Satellite Situation Reports,¹ in units of days. Therefore, the amplitudes of the off resonant mean anomaly perturbations, with C and BP_R in deg./day² and days are:

$$A = \frac{C(BP_R)^2}{720\pi} ,$$

radians. Estimating the mean along track perturbation amplitude (ΔR) as aA , this amplitude is given as

$$\Delta R = \frac{C(BP_R)^2 (6378)a}{720\pi} = 2.82 C(BP_R)^2 a , \text{ km.} \quad (5)$$

with C in deg/day², BP_R in days and, a, the near resonant semimajor axis, in earth radii.

USE OF THE GRAPHS

Figures 1 and 2 (extended from those in Wagner²) in conjunction with C values from Equation 4a and use of Equation 5 enable one to quickly calculate the off resonant perturbations on any satellite from a wide variety of given elements. Alternately, with beat period and C values, Figure 4 may be used instead of Equation 5.

For example, with mean period data from the Satellite Situation Reports,¹ the resonant period may be estimated from Figure 1 as a function of the orbit

inclination and eccentricity. Mean perigee and apogee heights, also listed in these reports, can be used in Figure 3 to find the orbit eccentricity. The off resonant period distance (resonant period minus actual period) is then used to find the beat period in Figure 2. The beat period is then combined with an estimate of the maximum $\ddot{M}_{\ell_{mpq \text{ resonant}}}$ or C value from Equation 4a or appropriate graphs, to find the dominant resonant along track perturbation from Equation 5 or Figure 4.

If the "mean" mean motion \bar{n} (revs./day) is given instead of the period, as it is in the Smithsonian Astrophysical Observatory Catalogues, this may be converted to the mean period in minutes: $P = 1440/\bar{n}$. Or, alternately, the graphs in Figure 1 give the resonant mean motion using the determining line and the rightmost scale. The actual mean period and the actual "mean" mean motion (according to the formula above) are in horizontal alignment between the left and rightmost scales in Figure 1. Thus, if the off resonant period distance is desired, the resonant period is determined in the usual way from Figure 1 and the actual mean period is found by the simple horizontal alignment of the left and rightmost scales.

If mean semimajor axis \bar{a} is given instead of period, the resonant semimajor axis may be found from the determining lines and the inner right hand scales in Figure 1. Then the off resonant period distance can be approximated upon differentiating Kepler's period law: $\Delta P = 3P \Delta a / 2a$ (see Figure 5).

Having found the off resonant semimajor axis distance Δa from Figure 1, Figure 5 may be used to find the off resonant period distances for near commensurable orbits from $S = 1$ to 15 revolutions per day. The factors $3P/2a$ for these commensurabilities were taken from Figure 1 for exactly resonant orbits at $I = 40^\circ$ and $e = 0$. For actual near commensurable orbits, these factors will be adequately close to those given in Figure 5.

APPLICATION

As an example of the near resonance perturbation calculation for an existing satellite, there is the satellite 1961-15A (OMI 1) which has already been used for geodetic purposes and thus serves as a good calibration object. In the Satellite Situation Report of April 15, 1968,¹ the period of this 14 revs/day object (Transit 4A) is listed as 103.8 minutes. The orbit inclination is 66.8° . From Figure 1, the exact resonant period for this object is 101.85 minutes. The resonant period distance (ΔP) is therefore 1.95 minutes. From Figure 2, the indicated beat period is 3.9 days. It should be noted that the actual orbit period is greater than that for exact resonance. Therefore, due to drag, Transit 4A will show a slowly lengthening beat period and greater resonance perturbations as time goes on. In 1962, the theoretical beat period was 3.7 days according to Anderle.⁵ Since it is a $q = 0$ harmonic term which is dominant on this nearly circular orbit satellite, the beat period calculated from Figures 1 and 2 is exact. Where the q index of the dominant harmonic term is not zero and $\dot{\omega}$ is appreciable,

the dominant beat period will be somewhat different than that estimated from Figures 1 and 2. (See Wagner²).

According to Anderle,⁵ in 1962 the resonance perturbations on this object (1961-15A) were about 100 meters in amplitude (due to $J_{15,14}$). Using a beat period of 3.9 days and a dominant C value of 0.67×10^{-3} deg/day² for the effect of the dominant $V_{15,14,7,0}$ geopotential term, as found in Douglas and Palmiter,⁴ Equation 5 gives:

$$\Delta R_{15,14,7,0} = 35 \text{ meters}$$

for 1961-15A. A large part of the discrepancy between this calculated perturbation and the actually observed one is due to the fact that the C value graphs in Douglas and Palmiter⁴ are based on a rule of thumb estimation of $J_{\ell m}$ for ℓ, m indices above 12, 12 which gives $J_{15,14}$ about 4.4 times smaller than $J_{15,14}$ measured by Anderle⁵ from this object. In the extended, fine spectrum study of these existing orbits, a more realistic geoid with all coefficients through $J_{15,15}$ will be used.

RESONANT SATELLITE SURVEY RESULTS

A total of 85 objects in near resonant orbits have been examined in detail according to their mean orbital elements (period, inclination, apogee and perigee heights) as listed in the April 15, 1968 Satellite Situation Report of the Goddard Space Flight Center's Operation Control Center¹ (see Table 1). These elements have been used, in conjunction with Figures 1, 2, and 3 of this report, to estimate the off resonant beat periods for the objects.

Using previously published resonant acceleration data in Douglas and Palmiter⁴ based on a Geoid through $J_{12,12}$ by Kaula,¹⁶ in conjunction with the estimates of the off resonant beat periods, amplitudes of along track perturbations due to the dominant resonant harmonic effects have also been calculated. These calculations calibrate rather well with actual near resonant perturbations measured from geodetic satellite orbits. All existing near resonant geodetic satellite orbits (as of April 1968) studied by Gaposchkin (1966),⁹ Yionoulis (1965),⁷ Anderle (1965),⁵ Guier and Newton (1965),⁶ Kaula (1968)¹⁷ and Wagner (1968)¹⁰ have been examined and evaluated in this manner for their principal off or near resonant effects. In addition all the objects listed in Strange, et al.,¹⁴ as being in reasonably good resonant orbits for geodetic purposes, have also been evaluated. Some of these include those already tracked and analyzed for geodetic effects.

In addition to the objects listed in the above sources, about 30 new ones have been found in the Satellite Situation Report¹ which should show geodetically significant near resonant perturbations when their orbits are analyzed for these effects.

The list of all the evaluated objects is presented in Table 1. It should not be considered exhaustive of the interesting near resonant orbits that exist about the earth. The selection of "new" objects was based primarily on finding the orbits with largest estimated perturbations with respect to each of the one day commensurabilities from $S = 1$ to 15 revs/day. Where more than one object

was found in essentially the same orbit, the orbit closest to resonance was chosen for evaluation. Since perturbation magnitude was the chief criteria within each commensurability, interesting near resonant objects of smaller perturbation but varied inclination and eccentricity (important to satellite geodesy) may have been missed in this survey. It should also be kept in mind that these examined orbits are only for April 1968, and that due to air drag, luni-solar gravity, radiation pressure or even controlled or uncontrolled satellite outgassing, many of the listed orbits and their resonance characteristics will change appreciably with time. In the past there may have been other listed or unlisted satellites which suffered even more interesting and unexamined geodetic effects than those checked in Column ① of Table 1. In the future the situation may change also for some of the listed objects here. To aid in follow up and follow back investigations of these objects, the beat period (Column ⑨ in Table 1) has a + sign superscript if the orbits mean distance is above the exact resonant mean distance, and a - sign if it is below. For the low perigee, near circular orbit satellites, the higher energy orbits should show stronger resonance effects in the future. Similarly, the lower energy ones (with respect to exact resonance) should have shown stronger effects in the past.

Up to five "new" (presumably unanalyzed) near resonant orbits with the strongest estimated perturbations for each commensurability have been selected from Table 1 as worthy of further tracking investigation for geodetic purposes. They are the ✓ checked (in Column ①) objects, and their gross perturbation

characteristics for resonant satellite geodesy are summarized in the abstract.

The sensitive, resonant, harmonics on these (and the other) listed orbits are found in Column (11) of Table 1. They represent the harmonics which may cause a perturbation of at least 10% of the dominant perturbation in Column (10). Where the orbit inclination is much over 90° , no good estimate of \ddot{M}_{\max} from the graphs in Douglas and Palmiter⁴ is available. The perturbations for these orbits (with a ? question mark in Columns (10) and (11)) are estimated at 1/2 the maximum perturbation in the range $0 < I < 90^\circ$. More information on the physical characteristics and tracking capabilities of these objects are to be found in papers by King-Hele and his associates.¹⁸

The only resonant orbits not surveyed here are the maneuverable 24 hour communications satellites in NASA's Syncom and ATS series, and COMSAT's Intelsat series; 1963 31A (SYNCOM 2), 1964 47A (SYNCOM 3), 1966 110A (ATS 1), 1967 111A (ATS 3), 1965 28A (Early Bird), 1967 01A (Intelsat 2 F-2), 1967 26A (Intelsat 2 F-3) and 1967 94A (Intelsat 2 F-4). The orbits of these satellites are generally in the libratory regime of deep resonance and are presently being studied in detail for geodetic effects.¹⁹

Summarizing, the results of a survey of about 1000 existing satellite orbits of the earth for application to resonant satellite geodesy show:

1. Thirty-six orbits (previously unanalyzed for the long term amplified along track oscillations of resonance) whose detailed tracking analysis should contribute significantly to the refinement of knowledge of the earth's longitude gravity field.

2. Of the thirty-six new orbits, the most favorable for satellite geodesy are probably five 12 hour orbits (2 revs/day) of communications-satellites of the United States (INTELSAT 2 F-1) and the U.S.S.R. (in the Molniya and Cosmos series). These orbits should be suffering along track perturbations of from 7000 to 400,000 kilometers with periods of the order of a year and more. Study of these perturbations should considerably improve knowledge of low degree and order longitude coefficients in the geopotential.
3. A good variety of inclinations and eccentricities for fairly strong new resonant orbits exist near commensurabilities of 12 and 14 revolutions per day. About 10 of these orbits should be suffering resonant perturbations of from 0.5 to 80 kilometers along track with periods of from 15 to 250 days. Study of these should greatly improve the definition of many specific gravity harmonics of order 12 and 14.
4. A scarcity of good resonant orbits for satellite geodesy exists at commensurabilities of 4, 5, 6, 7, 9 and 10 revolutions per day. It is hoped that in the future, some space missions close to these commensurabilities can be altered to bring them even closer so as to improve their usefulness as measuring probes of the earth's gravity field.

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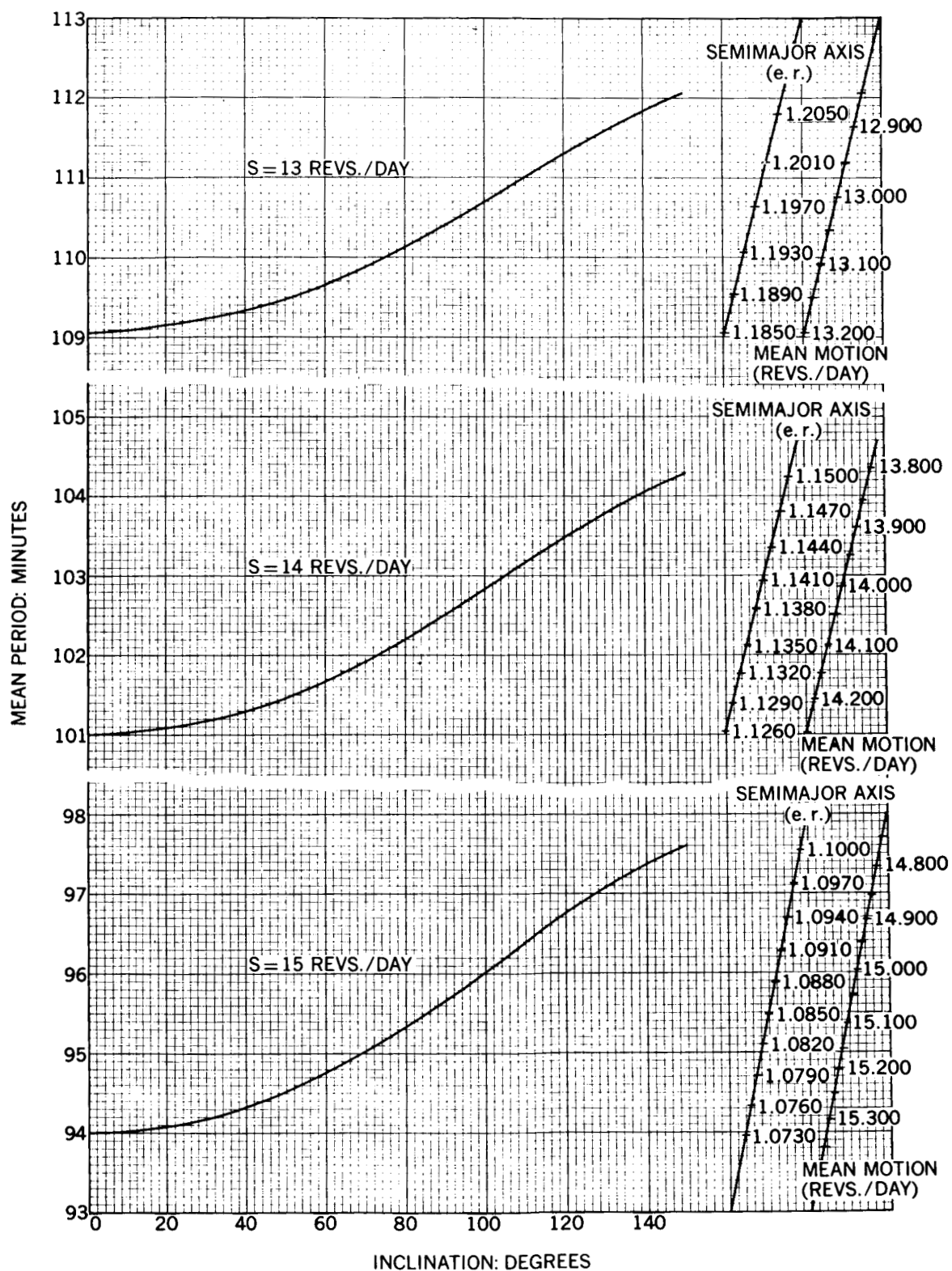


Figure 1-Resonant periods, semimajor axes and mean motions for commensurable earth satellite orbits.

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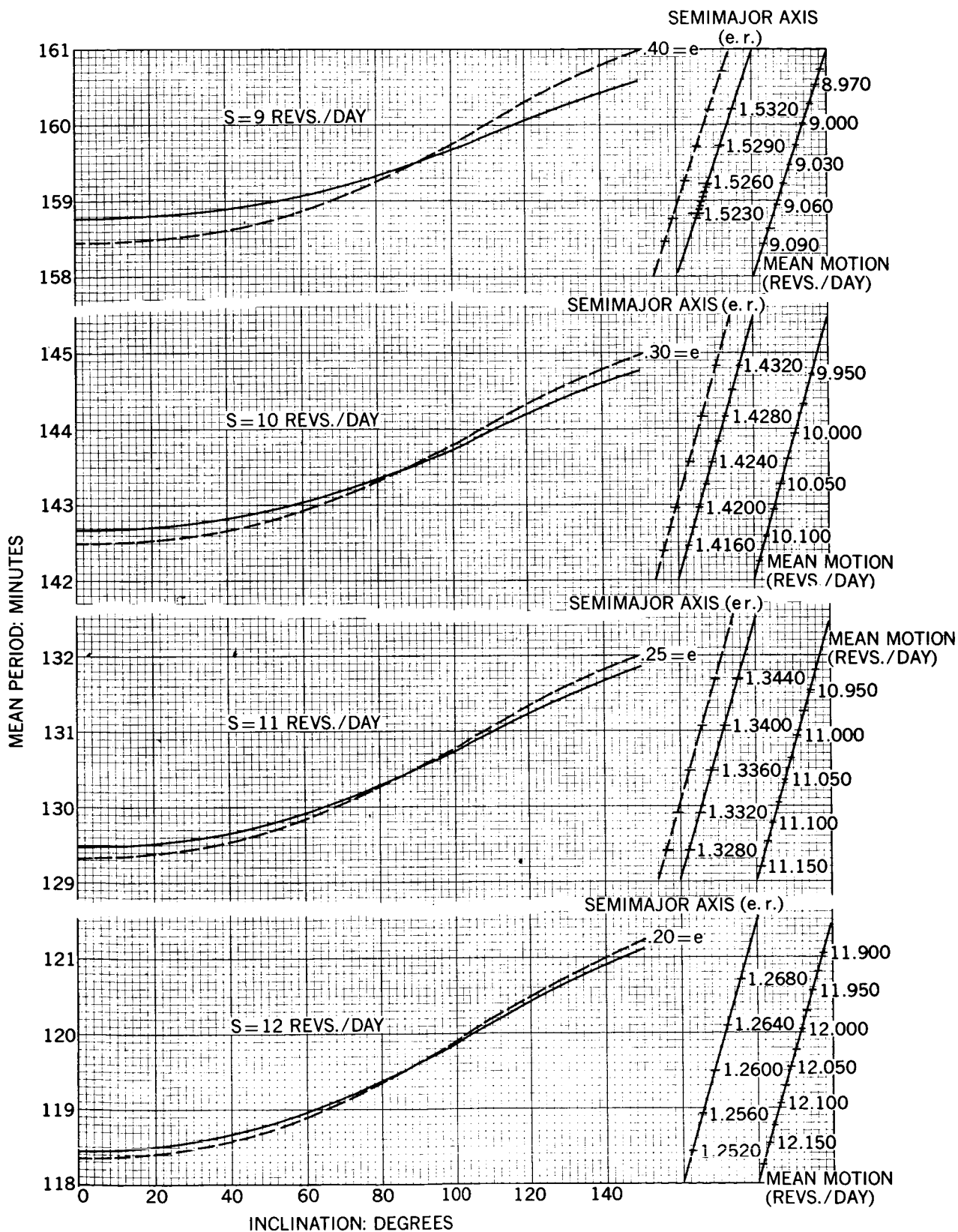


Figure 1-Resonant periods, semimajor axes and mean motions for commensurable earth satellite orbits.

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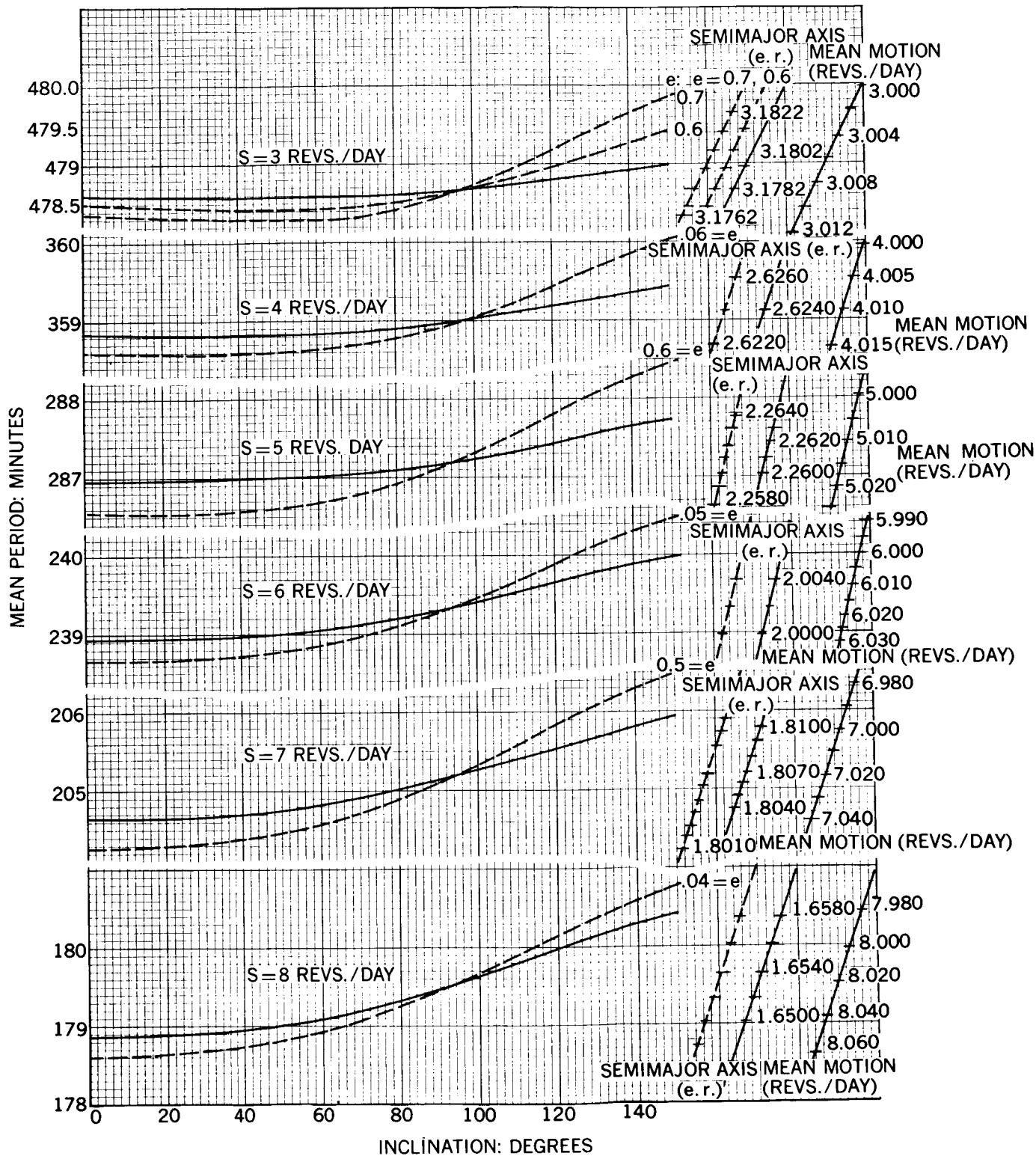


Figure 1-Resonant periods, semimajor axes and mean motions for commensurable earth satellite orbits.

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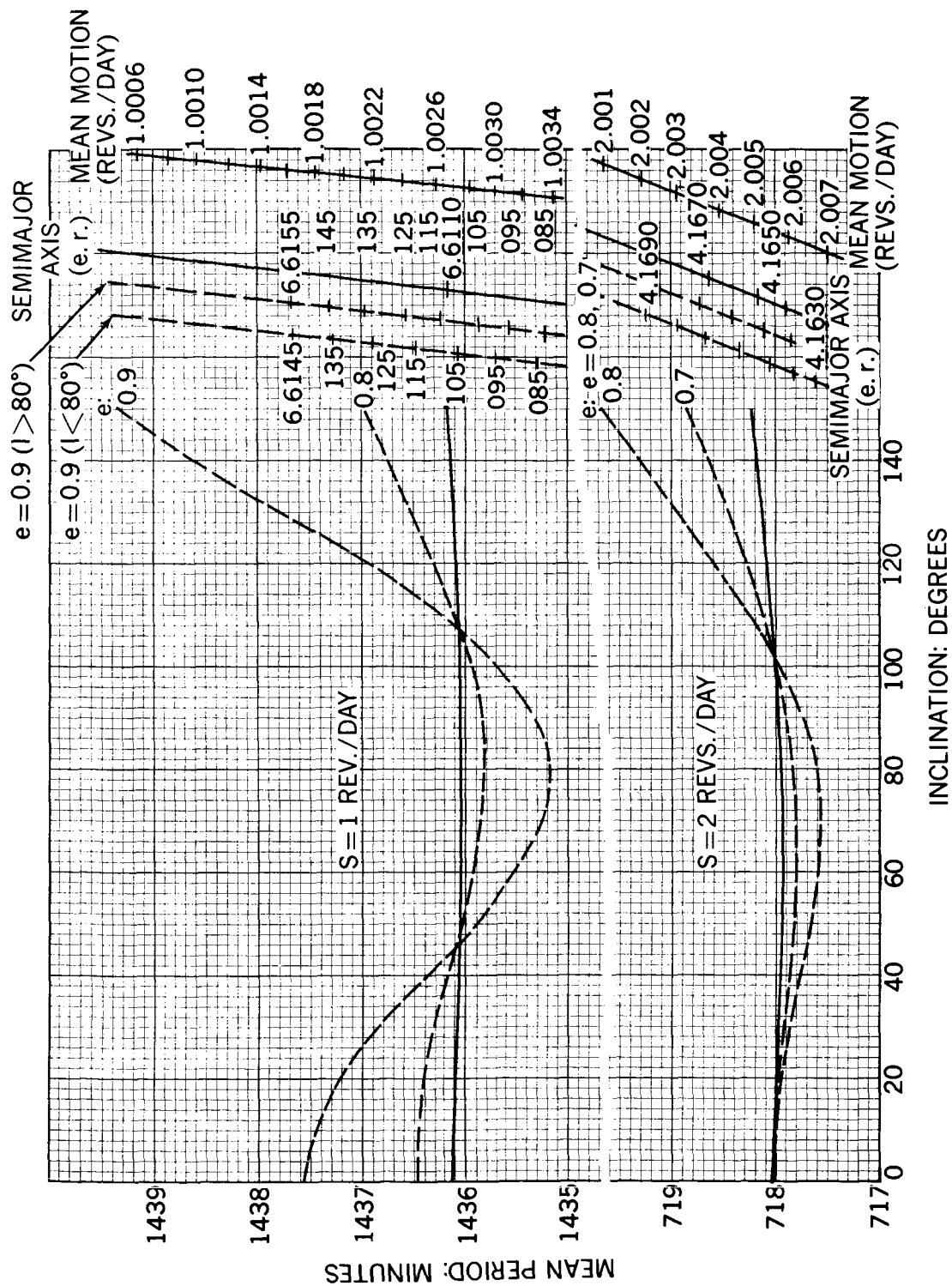


Figure 1-Resonant periods, semimajor axes and mean motions for commensurable earth satellite orbits.
Sheet 4 of 4

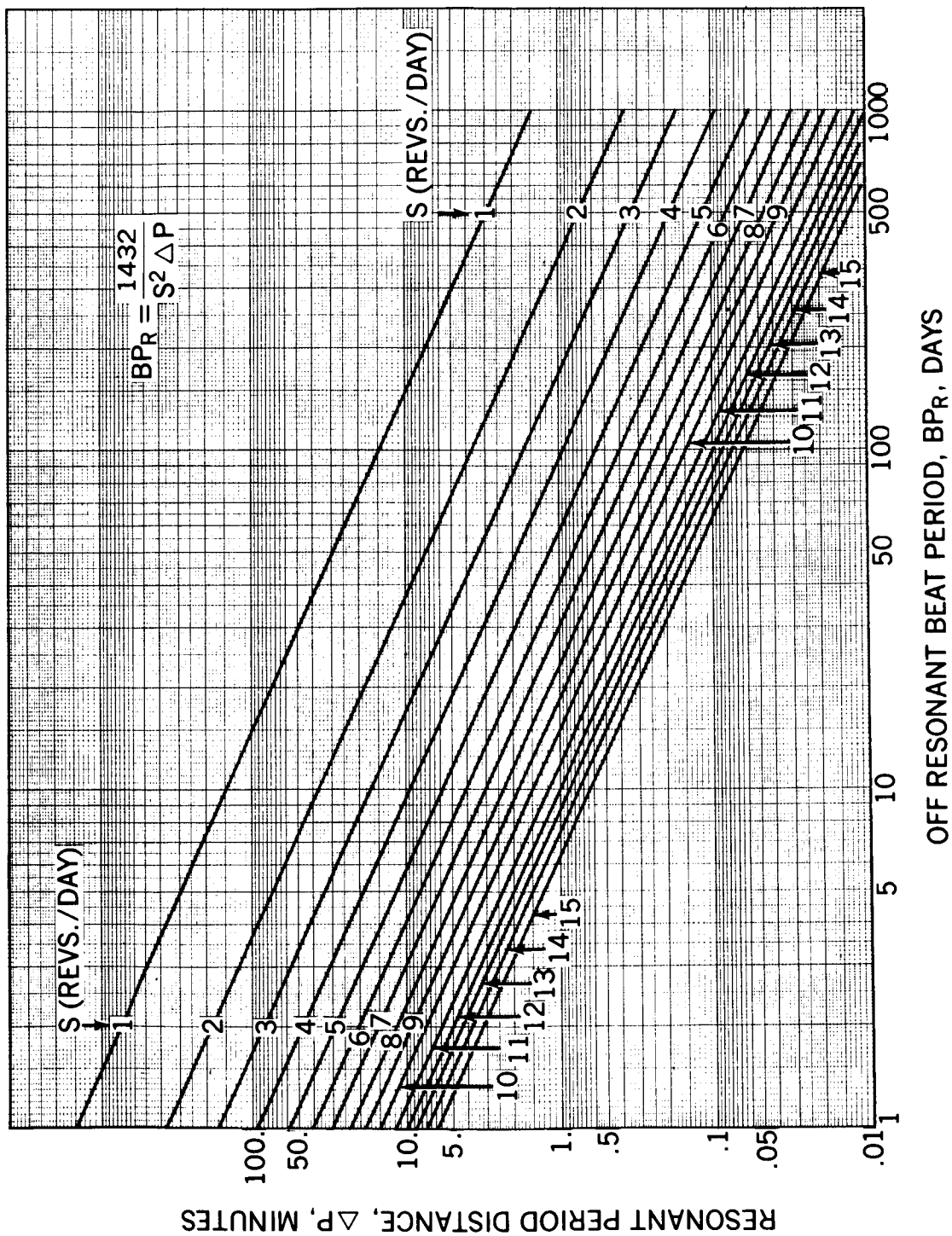


Figure 2-Off resonant beat period as a function of period distance from resonance for near commensurable earth satellite orbits.

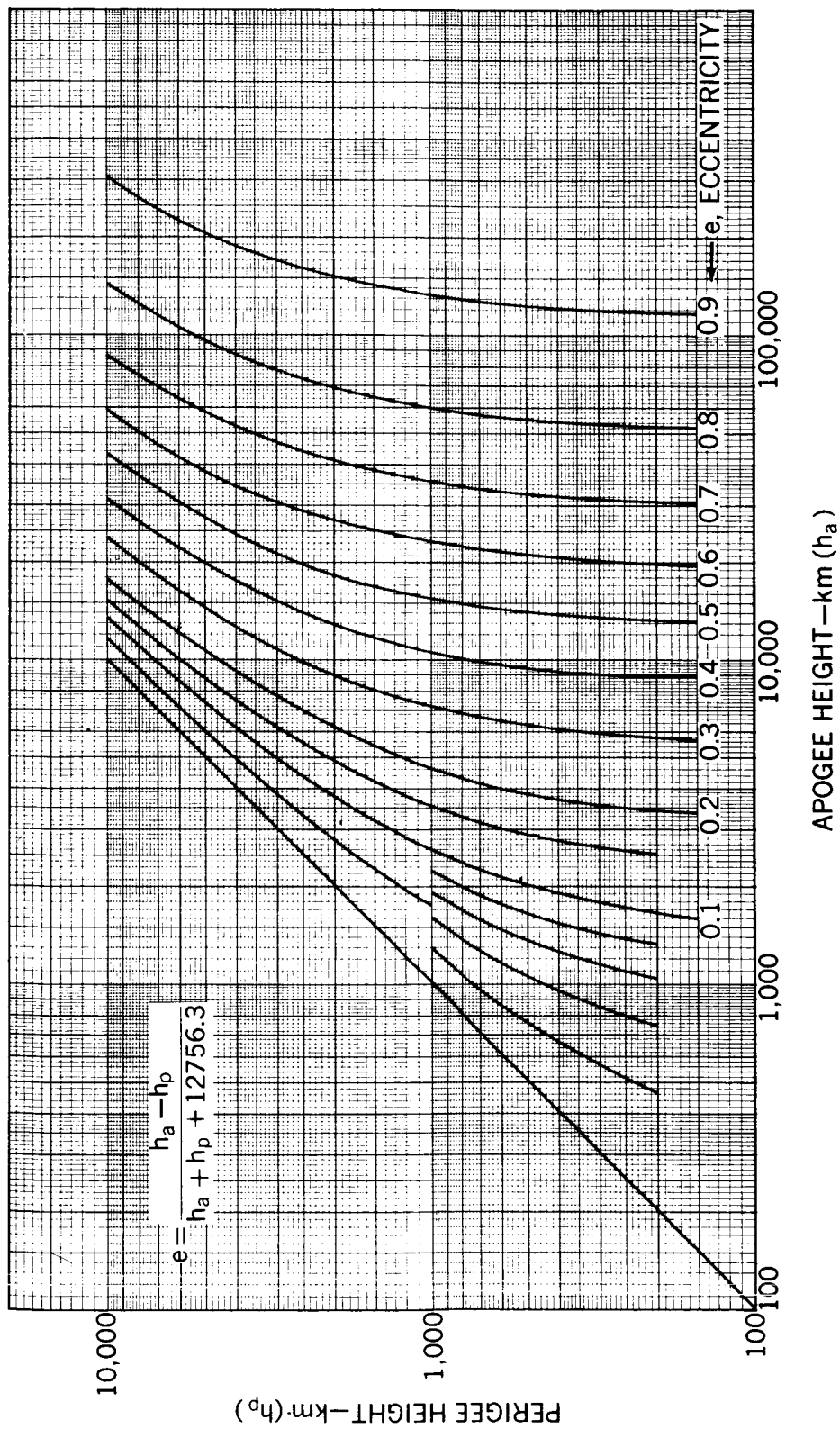


Figure 3- Eccentricity as a function of perigee and apogee heights.

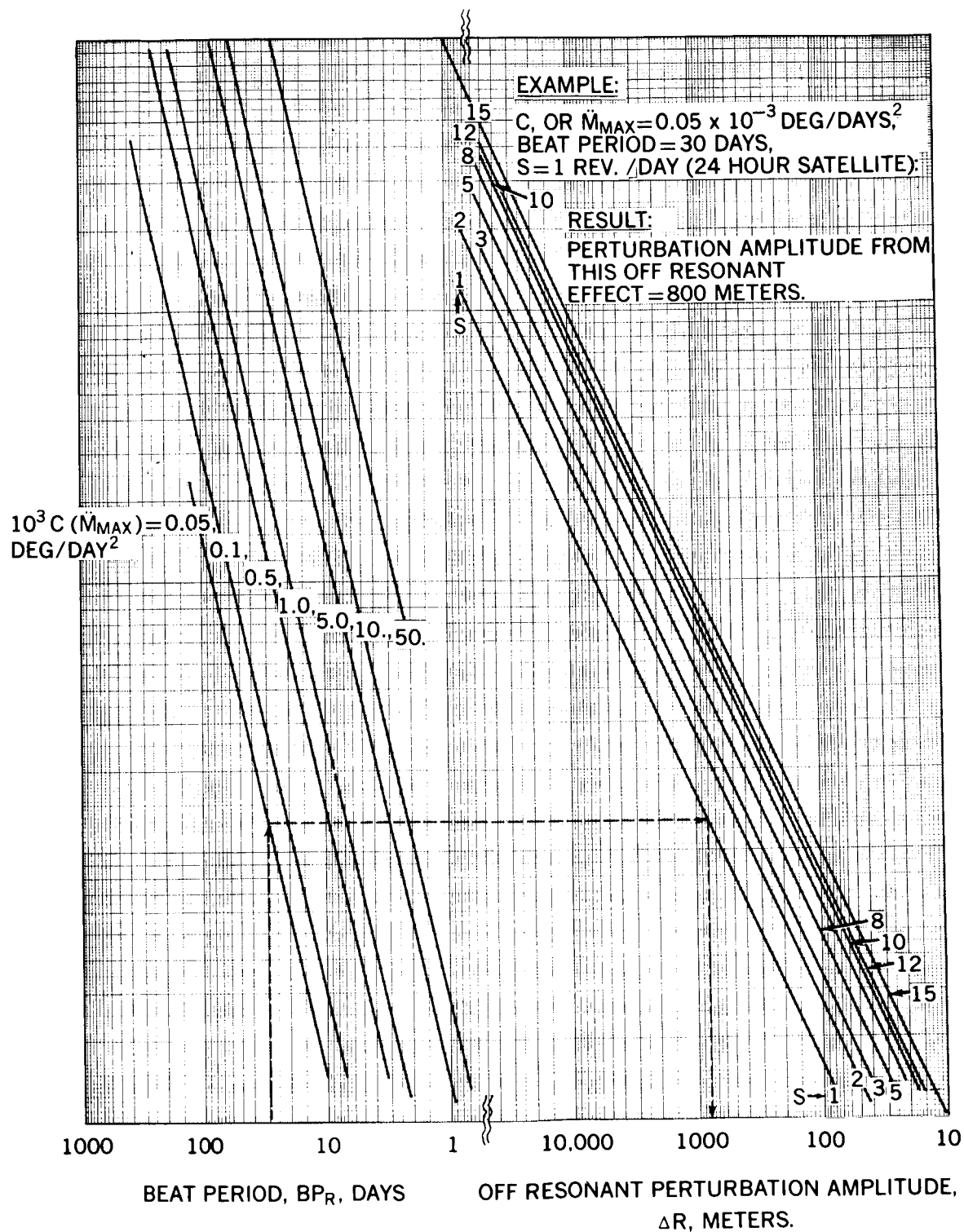
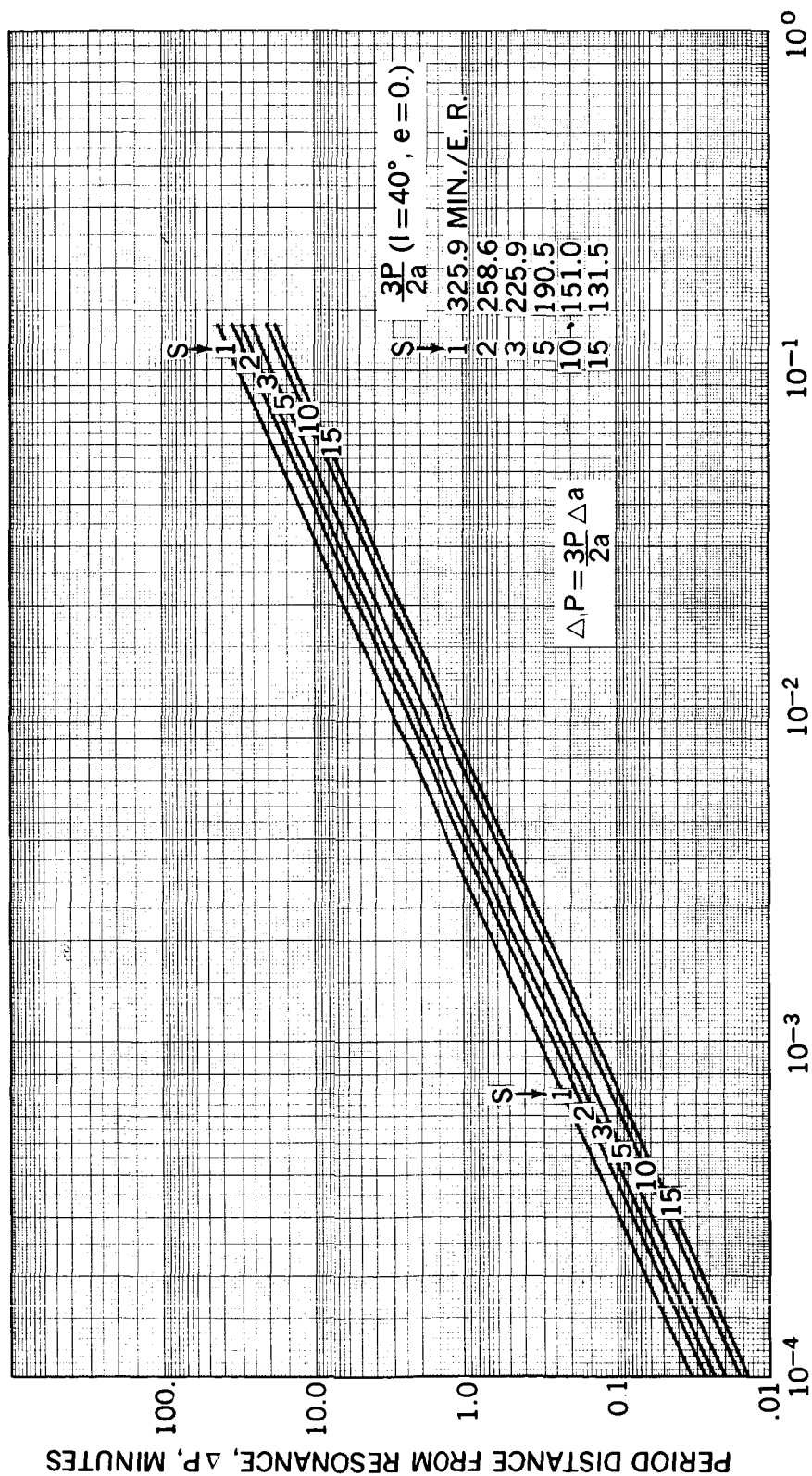


Figure 4-Off resonant along track perturbation from beat period and maximum mean anomaly acceleration.



Δa (SEMIMAJOR AXIS DISTANCE FROM RESONANCE), EARTH RADII

Figure 5-Off resonant period distance as a function of semimajor axis distance from resonance.

TABLE 1
SURVEY OF EXISTING NEAR RESONANT EARTH SATELLITE ORBITS.

①	SATELLITE	MEAN MOTION	PERIOD P	SEMI-MAJOR AXIS	NATION	ECCEN-TRICITY	PERIGEE ALTITUDE	⑨	⑩	⑪	NOTES: SATELLITE CODE NAME; GEODETIC USE; COMMENTS
		n, s (REV/S DAY)	(MIN.)	a (e. r.)	I (°)	e	hp (km)	RESONANT PERTURBATIONS			
								RESONANT PERIOD (DAYS)	DOMINANT AMPLITUDE (METERS)	SENSITIVE HARMONICS (H _{f, j})	
✓	1968 015B	15	95.1	1.0624	70.9	.06	267	200.4	134,000	H _{2,1} ; l=15, 17, 19	COSMOS 200 —; —; DEEPLY RESONANT
✓	1968 006A	15	95.1	1.0622	74.0	.00	516	125.7	61,700	H _{2,1} ; l=15, 17, 19	
✓	1968 006C	15	95.3	1.0639	74.0	.01	503	43.4	7,300	H _{2,1} ; l=15, 17, 19	
✓	1962 25C (A ALPHA 3)	14	101.6	1.1312	58.2	.04	600	250.7	75,500	H _{2,1} ; l=15, 16, 17	RESEARCH SAT. FOR GEOPHYSICS: SAO POLYOT 1 TRANSIT 2A
✓	1968 028B	14	102.3	1.1353	81.0	.10	196	150.4	28,600	H _{2,1} ; l=14-18	
✓	1963 26A	14	101.4	1.1294	49.7	.061	412	147.7	8,400	H _{2,1} ; l=15, 16, 17	
✓	1963 43A	14	101.4	1.1295	58.8	.08	335	30.7	1,150	H _{2,1} ; l=15, 16, 17	OVI-4 —; Y
✓	1960 07A (ETA 1)	14	101.5	1.1307	66.7	.04	615	21.7	940	H _{2,1} ; l=14, 15, 16, 17	
✓	1962 02B (BETA 2)	14	101.3	1.1285	48.1	.02	700	47.7	650	H _{2,1} ; l=15, 16, 17	
✓	1966 25A	14	104.0	1.1488	144.5	.01	887	50.7	290.7	H _{2,1} ; l=14-17	ECHO 1; —; BALLOON, DECAYED, MAY 1968
✓	1966 25D	14	104.0	1.1487	144.5	.01	886	50.7	290.7	H _{2,1} ; l=14-17	
✓	1964 26A	14	103.0	1.1415	90.5	.01	885	16.5*	260	H _{2,1} ; l=14-17	
✓	1963 54C	14	101.0	1.1265	58.4	.02	697	12.7	174	H _{2,1} ; l=15, 16, 17	GREB —; SAO
✓	1960 09A (OTA 1)	14	101.7	1.1321	47.2	.02	730	25.4	159	H _{2,1} ; l=15, 16, 17	
✓	1959 09A (OTA 1)	14	101.0	1.1265	50.3	.05	551	16.7	114	H _{2,1} ; l=15, 16, 17	
✓	1965 16A	14	105.4	1.1449	70.0	.00	907	5.2*	59	H _{2,1} ; l=15, 17	TRANSIT 4A; SAO, NWL, Y, K, APL INUN-SR-3; SAO
✓	1964 01A	14	103.4	1.1446	69.9	.002	910	5.2*	58	H _{2,1} ; l=15, 17	
✓	1964 31A	14	101.5	1.1306	66.8	.00	828	5.6*	43.7	H _{2,1} ; l=15, 17	
✓	1961 15A (OMI 1)	14	103.8	1.1471	66.8	.008	883	3.9*	35	H _{2,1} ; l=15, 17	EXPLORER 20 EXPLORER 24 EXPLORER 22; SAO, K OGO 2; SAO
✓	1961 15B (OMI 2)	14	103.8	1.1473	66.8	.008	884	3.9*	35	H _{2,1} ; l=15, 17	
✓	1964 51B	14	103.8	1.1474	79.9	.01	867	4.8*	30	H _{2,1} ; l=14, 15, 17	
✓	1964 51A	14	103.9	1.1480	79.9	.01	867	4.6*	28	H _{2,1} ; l=14, 15, 17	EXPLORER 20 EXPLORER 24 EXPLORER 22; SAO, K OGO 2; SAO
✓	1964 76A	14	104.5	1.1530	81.3	.07	505	3.6*	15	H _{2,1} ; l=14-18	
✓	1964 64A	14	104.7	1.1541	79.6	.012	887	3.0*	14	H _{2,1} ; l=14, 15, 17	
✓	1965 81A	14	103.6	1.1463	87.3	.075	419	6.5*	13	H _{2,1} ; l=14-18	DIADEME 1 EXPLORER 1
✓	1965 109A	14	105.0	1.1562	89.0	.01	909	3.3*	5	H _{2,1} ; l=14, 15, 16, 17	
✓	1966 31B	14	100.7	1.1246	35.0	.00	786	14.7	3	H _{2,1} ; l=15, 17	
✓	1967 11A	14	104.1	1.1492	39.9	.06	566	3.3*	1	H _{2,1} ; l=15, 16, 17	DIADEME 2 EXPLORER 8 EXPLORER 27
✓	1958 01A (ALPHA 1)	14	100.1	1.1194	33.1	.07	339	7.7	0	H _{2,1} ; l=15, 16, 17	
✓	1968 011B	13	109.3	1.1875	74.0	.00	1185	12.7	241	H _{2,1} ; l=13, 15, 17	
✓	1965 27C	13	111.3	1.2026	90.2	.00	1262	9.5*	187	H _{2,1} ; l=13, 15, 17, 19	GEOS 2 ROCKET
✓	1965 27A	13	111.5	1.2033	90.2	.00	1271	8.4*	133	H _{2,1} ; l=13, 15, 17, 19	
✓	1968 002B	13	112.1	1.2081	105.8	.03	1083	7.7*	54.7	H _{2,1} ; l=13-18	
✓	1965 48A	13	106.9	1.1700	89.9	.01	1028	3.5*	19	H _{2,1} ; l=13, 14, 15, 17, 19	EXPLORER 19; SAO SN-39; APL ANNA 1B; SAO, NWL, K, APL —; SAO, APL, NWL, Y
✓	1963 53A	13	113.1	1.2152	78.7	.08	767	3.3*	18	H _{2,1} ; l=13-19	
✓	1963 38C	13	107.3	1.1731	89.9	.00	1072	2.8*	17	H _{2,1} ; l=13, 15, 17	
✓	1962 60A (B MU 1)	13	107.9	1.1770	50.1	.01	1075	5.6*	16	H _{2,1} ; l=13, 15, 17	DIADEME 2 EXPLORER 8 EXPLORER 27
✓	1963 49B	13	107.1	1.1715	89.9	.005	1070	2.6*	14	H _{2,1} ; l=13, 15, 17, 19	
✓	1967 14A	13	110.1	1.1929	39.4	.09	587	11.5*	11	H _{2,1} ; l=13-18	
✓	1960 14A (XI 1)	13	111.7	1.2045	49.9	.11	416	4.4*	9	H _{2,1} ; l=13-18	TIROS 9 D-1A
✓	1965 32A	13	107.8	1.1764	41.1	.03	941	6.7	4	H _{2,1} ; l=13-18	
✓	1965 89C	12	119.1	1.2578	59.3	.07	1135	67.7	11,400	H _{2,1} ; l=12-17	
✓	1961 04B (DELTA 2)	12	118.5	1.2528	38.8	.12	633	67.7	965	H _{2,1} ; l=12-17	ALOUETTE 2
✓	1965 04A	12	119.1	1.2577	96.4	.11	708	15.7	950.7	H _{2,1} ; l=12, 13, 17	
✓	1966 13A	12	118.5	1.2529	34.0	.14	508	100.7	775	H _{2,1} ; l=12-17	
✓	1960 09E (OTA 5)	12	118.4	1.2524	47.2	.01	1536	29.7	532	H _{2,1} ; l=13, 15, 17	EXPLORER 29 (GEOS 1); SAO, K ECHO 1 ROCKET; SAO, K
✓	1965 98A	12	121.1	1.2720	79.8	.15	506	6.4	254	H _{2,1} ; l=12, 13, 17	
✓	1966 13B	12	118.4	1.2527	34.0	.14	500	50.7	194	H _{2,1} ; l=12-17	
✓	1965 89A	12	120.3	1.2658	59.3	.072	1119	7.3*	124	H _{2,1} ; l=12-17	EXPLORER 29 (GEOS 1); SAO, K ECHO 1 ROCKET; SAO, K
✓	1960 09B (OTA 2)	12	118.3	1.2498	47.2	.012	1509	13.5*	124	H _{2,1} ; l=12, 13, 15, 17	
✓	1965 63B	12	122.2	1.2792	69.2	.08	1140	3.3*	41	H _{2,1} ; l=12, 13, 14, 17	
✓	1965 63A	12	122.2	1.2791	69.2	.08	1138	3.3*	41	H _{2,1} ; l=12, 13, 14, 17	EGRS 5 —; SAO; DEEPLY RESONANT
✓	1959 01B (ALPHA 2)	11	129.5	1.3291	32.9	.183	554	300.7	12,100	H _{2,1} ; l=11-17	
✓	1963 25B	11	130.9	1.3396	82.1	.21	342	20.4	8,250	H _{2,1} ; l=11, 12	
✓	1959 07A (ETA 1)	11	129.6	1.3300	33.3	.190	514	120.4	2,050	H _{2,1} ; l=11-17	VANGUARD 3; SAO
✓	1966 52A	10	143.1	1.4211	40.8	.22	644	41.4	1,600	H _{2,1} ; l=10-15	
✓	1966 52B	10	143.1	1.4207	40.8	.22	644	41.4	1,600	H _{2,1} ; l=10-15	
✓	1965 08A	10	145.3	1.4355	32.1	.00	2763	6.4	8	H _{2,1} ; l=11, 13, 15	TELSTAR 1; SAO MIDAS 3
✓	1962 29A (A EPS 1)	9	157.7	1.5164	44.8	.25	940	16.7	877	H _{2,1} ; l=9, 10, 11, 13	
✓	1961 18A (SIGMA 1)	9	161.4	1.5401	91.1	.01	3346	625.7	605	H _{2,1} ; l=9, 11	
✓	1966 00C	9	161.5	1.5408	84.6	.28	667	8.4*	605	H _{2,1} ; l=9, 10, 11	MIDAS 4; SAO, K
✓	1966 00B	9	161.9	1.5430	84.6	.28	725	7.7	420	H _{2,1} ; l=9, 10, 11	
✓	1965 34A	9	157.0	1.5113	32.1	.05	2781	10.7	82	H _{2,1} ; l=9, 10, 11, 13, 15	
✓	1961 28A (A DELTA 1)	9	165.9	1.5685	96.8	.01	3503	2.8*	45	H _{2,1} ; l=9, 11	PAGEOS 1; —; BALLOON: A/M = 130 cm ² /gm.
✓	1966 00A	9	155.2	1.4998	35.1	.31	214	5.4	34	H _{2,1} ; l=9-15	
✓	1966 56A	8	179.7	1.6542	84.9	.20	2114	65.7	60,100	H _{2,1} ; l=8, 9, 10	
✓	1966 56B	8	181.1	1.6631	86.9	.01	4175	14.7	2,820	H _{2,1} ; l=8, 9, 10	RELAY 1 OVI-14 TELSTAR 2
✓	1962 68B (B UPS 2)	8	184.8	1.6852	47.5	.28	1331	3.9*	102	H _{2,1} ; l=8-12	
✓	1962 68A (B UPS 1)	8	185.0	1.6865	47.5	.28	1331	3.8*	97	H _{2,1} ; l=8-12	
✓	1966 26B	7	207.8	1.8228	100.0	.40	560	11.7	430.7	H _{2,1} ; l=7, 8, 9?	INTELSAT 2 F-1; —; PROBABLY LIBRATING MOLNIYA 4; —; PROBABLY LIBRATING COSMOS 41 ROCKET BODY; —; DEEPLY RESONANT MOLNIYA 6; —; DEEPLY RESONANT MOLNIYA 7; —; DEEPLY RESONANT MOLNIYA 1; W COSMOS 41; W COSMOS 174 MOLNIYA 5 MOLNIYA 3
✓	1963 13A	6	225.2	1.9228	42.7	.40	964	3.0*	102	H _{2,1} ; l=6-10	
✓	1965 34D	5	309.7	2.3776	32.2	.40	2773	2.4*	73	H _{2,1} ; l=5, 6, 8, 9; H _{11,10}	
✓	1963 31B	4	375.4	2.7028	32.7	.61	262	5.3*	291	H _{2,1} ; l=4-7	DIADEME 2 EXPLORER 8 EXPLORER 27
✓	1966 110B	3	494.1	3.2459	31.0	.88	168	10.3*	3,810	H _{2,1} ; l=3-5, 4, 5	
✓	1966 96A	2	717.9	4.1640	17.4	.63	3197	3400.7	390,000,000	H _{2,1} ; l=2-5; H _{1,1} ; l=4-7	
✓	1966 92A	2	716.9	4.1605	64.9	.74	269	480.7	25,400,000	H _{2,1} ; l=2-5; H _{1,1} ; l=4-7	DIADEME 2 EXPLORER 8 EXPLORER 27
✓	1964 49E	2	716.4	4.1584	68.8	.70	1531	275.7	8,800,000	H _{2,1} ; l=2, 3, 5; H _{1,1} ; l=4-7	
✓	1967 95A	2	716.2	4.1577	64.9	.74	332	250.7	8,800,000	H _{2,1} ; l=2-5; H _{1,1} ; l=4-7	
✓	1967 101A	2	716.2	4.1577	64.8	.73	527	250.7	8,800,000	H _{2,1} ; l=2-5; H _{1,1} ; l=4-7	DIADEME 2 EXPLORER 8 EXPLORER 27
✓	1965 30A	2	720.1	4.1727	65.3	.68	2021	145.4	2,370,000	H _{2,1} ; l=2, 3, 5; H _{1,1} ; l=4-7	
✓	1964 49D	2	714.6	4.1515	68.9	.71	1415	115.7	1,550,000	H _{2,1} ; l=2, 3, 5; H _{1,1} ; l=4-7	
✓	1967 82A	2	714.5	4.1513	64.9	.74	324	112.7	1,580,000	H _{2,1} ; l=2-5; H _{1,1} ; l=4-7	DIADEME 2 EXPLORER 8 EXPLORER 27
✓	1967 52A	2	710.8	4.1370	64.8	.72	901	52.7	250,000	H _{2,1} ; l=2-5; H _{1,1} ; l=4-7	
✓	1966 35A	2	705.6	4.1165	65.1	.71	1197	30.7	90,000	H _{2,1} ; l=2, 3, 5; H _{1,1} ; l=4-7	
✓	1964 06D	1	1384.0	6.4501	58.3	.75	3850	27.5	28,400	H _{2,1} ; l=12, 13, 41, 42, 43	DIADEMEOS